COT4210 Discrete Structures – Exam 2

Fall 2023

1. (30) Let *G* = (*V*, ∑, *R*, *S*) be a grammar with *V* = {Q, R, T}; ∑ = {**q**, **r**, **t**}; and the set of rules:

S à Q

* + 1. à **q** | R**q**T
    2. à **r** | **r**T | QQ**r** | 𝜀

T à **t** | S| **t**T

* 1. (25) Convert *G* to Chomsky Normal Form.

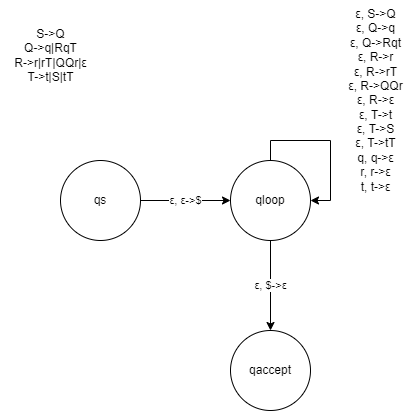
Add S and remove any :

Remove single rewrites:

Remove mixed/multiple terminals:

Remove long rewrites to get CNF:

* 1. (5) Convert the language of *G* to a PDA.



1. (20) Show that the class of context-free languages is closed under homomorphism.

Suppose we have a context free language and any homomorphism on terminal symbols , we can construct a grammar by replacing each terminal symbol with the result of that symbol in function h, so where

For example, language with

Then language

So if then

1. (15) Let 𝑅𝐸𝑃𝐸𝐴𝑇!"# = {〈𝑀〉|𝑀 is a DFA and for every 𝑠 ∈ 𝐿(𝑀), 𝑠 = 𝑢𝑣 where 𝑢 = 𝑣}. Show that 𝑅𝐸𝑃𝐸𝐴𝑇!"# is decidable.

Proof: Construction, assume which decides

on input <M>:

* WLOG, we receive DFA M
* Where for every 𝑠 ∈ 𝐿(𝑀)
  + where s can be split into even length substrings u and v
  + And u = v
  + Then we accept
* Otherwise reject.

This machine clearly accepts a DFA if it’s language only contains strings where and and rejects otherwise. Even if |s| is infinite it will still be decidable. For example, if s = a\* this would reject since a\* cannot be evenly split into u and v.

1. (15) Let 𝑆𝑈𝐵𝑆𝐸𝑇!"# = {〈𝑀$, 𝑀%〉 | 𝑀$ and 𝑀% are DFAs and 𝐿(𝑀$) ⊆ 𝐿(𝑀%)}. Show that 𝑆𝑈𝐵𝑆𝐸𝑇!"# is decidable.

Proof: Construction, assume which decides

on input <>:

* WLOG, we receive DFA 𝑀$ and
* Where:
  + Reject if only is infinite
  + Accept if every is also (so if 𝐿(𝑀$) ⊆ 𝐿(𝑀%))
  + Otherwise reject.

This machine clearly accepts a DFA if it’s 𝐿(𝑀$) ⊆ 𝐿(𝑀%) rejects otherwise.

1. (20) A Turing machine *M* is **verbose** on the input string *s* if, when finished computing on *s*, *M* does not shrink the input – that is, leaves at least as many non-blank characters on the left of the tape as *s* has total characters.

Let *VERBOSETM* = { <*M*, *w*> | *M* is a TM and is verbose on *s* }, and show that *VERBOSETM* is undecidable by reduction from *A*TM. Do not use Rice’s theorem.

Proof: If decides *VERBOSETM*. Assume BWOC that is decidable:

By definition of decidability let be its decider where:

* We receive string and Turing machine M
* We accept if M is a TM and is verbose on s
* We reject otherwise

Then we build a where:

* We mimic and we receive Turing machine M and string s
* We construct a new Turing machine that,
  + Receives string and TM M
  + Enumerator where w is the number of things on the tape
  + we reject if is greater than or equal to the length of string
    - This causes M to decide
  + Otherwise simulate the contrariwise of TM M on s
    - This mimics the opposite of M whether is accepts, or rejects. Also means that it mimics if it never halts

Interrogate using which causes to decide and creates a contradiction. Therefore *VERBOSETM* must be undecidable.